

## ПРВИ ГРАФИЧКИ РАД

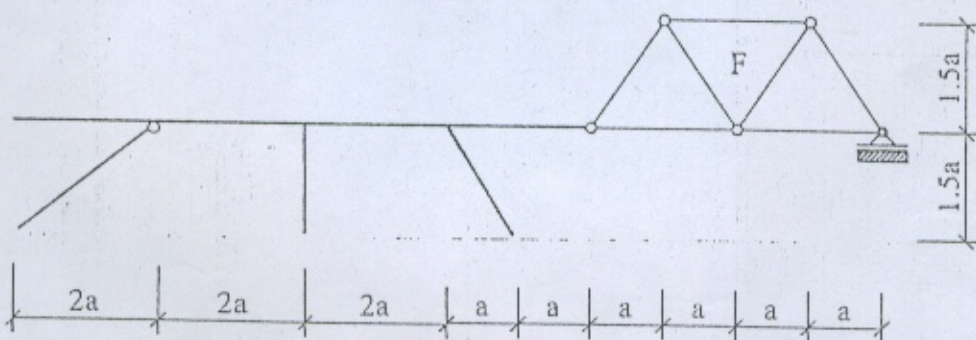
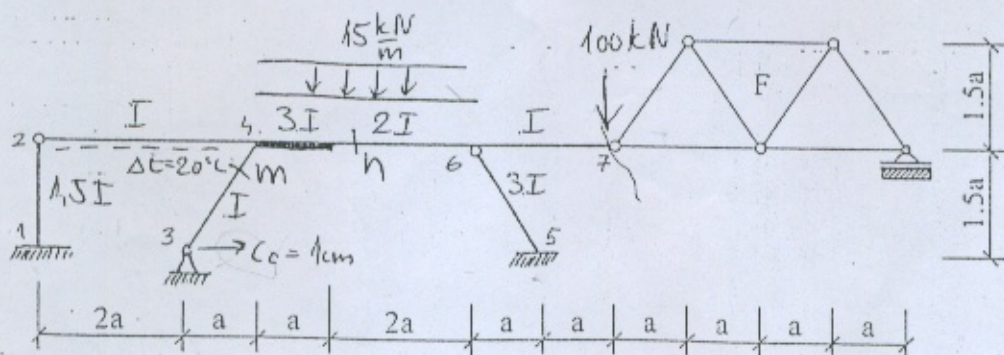
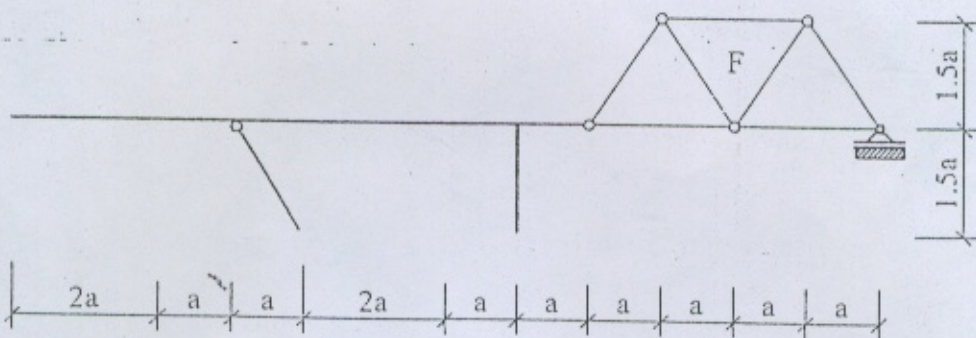
Име и презиме, број индекса:

ИВАН ТРКАЈАЦ 49/01

За задати носач:

- а) одредити статичку неодређеност,  
б) срачунати елементе матрице флексибилности  $\delta_{ij}$
- а) срачунати коефицијенте  $\delta_{i\alpha}$  услед задатог оптерећења и утицаја,  
б) одредити статички неодређене величине,  
в) нацртати дијаграм момената савијања, као и дијаграм тангенталних сила на означеним штаповима,
- а) услед задатог оптерећења и утицаја одредити задато генералисано померање,  $\Delta_{\eta}$   
б) нацртати утицајну линију за задату статичку величину,  $T_{\eta}$   
в) нацртати утицајну линију за задато генералисано померање.

ЗАНеМАРИТИ УТИЦАЈ НОРМАЛНИХ СИЛА НА ДЕФОРМАЦИЈУ НА ПУНОМ ДЕЛУ НОСАЧА.

 $I/F=1$ -за просте штапове $EI=10^5 \text{ kNm}^2$  $\alpha_t=10^{-5} \text{ 1/}^\circ\text{C}$  $h=1 \text{ m}$  $a=2 \text{ m}$ 

(+) 24.5.2004.  
B. Pajuk

Задатак задао

Уполичари М.



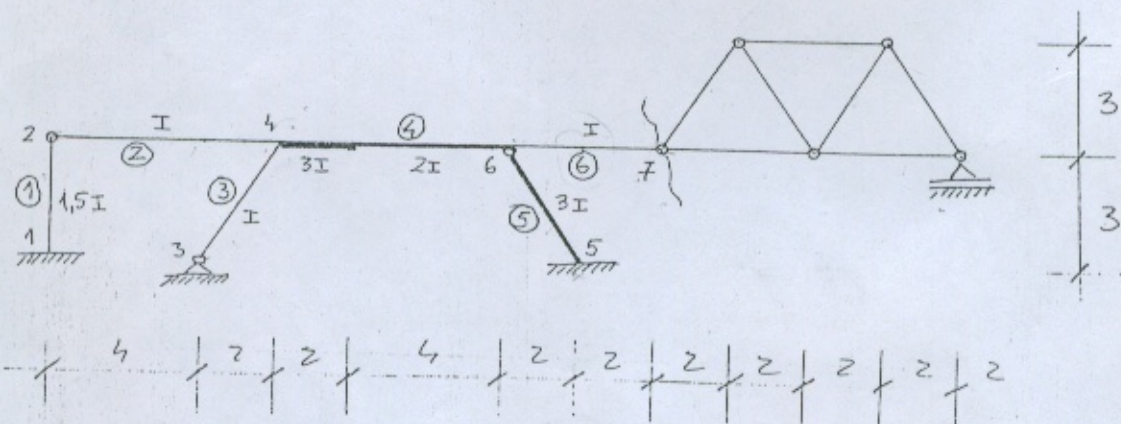
1.

a) ОДРЕЂИВАЊЕ СТАТИЧКЕ НЕОДРЕЂЕНОСТИ:

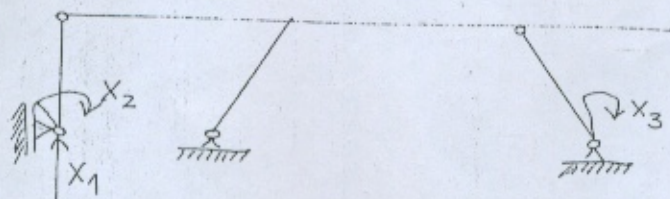
- БРОЈ ОСЛОНАЦА ....  $z_o = 6$
- БРОЈ УКЛЕШТЕЖА ....  $z_u = 2$
- БРОЈ КРУТИХ УГЛОВА ....  $z_k = 3$
- БРОЈ ШТАПОВА ....  $z_s = 6$
- БРОЈ ЧВОРОВА ....  $K = 7$

$$\mu = z_o + z_u + z_k + z_s - 2K = 6 + 2 + 3 + 6 - 2 \cdot 7 =$$

$$\Rightarrow \boxed{\mu = 3}$$

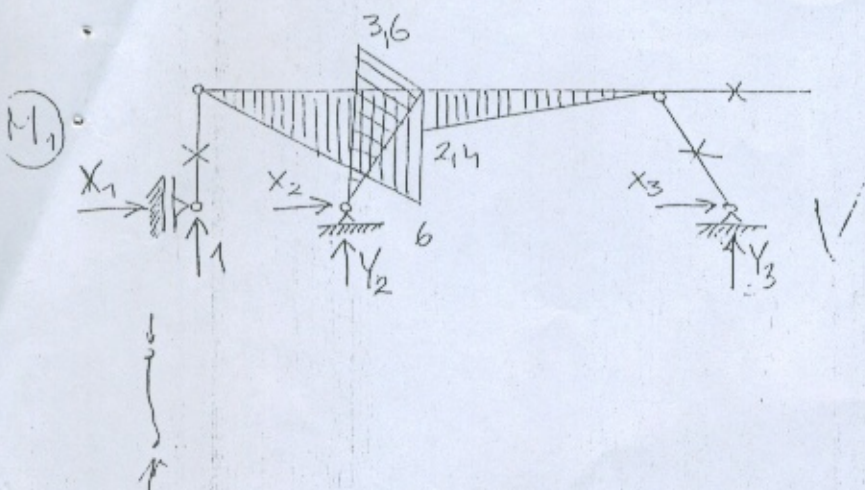


- ОСНОВНИ СИСТЕМ



114/02  
Петровић-Немања





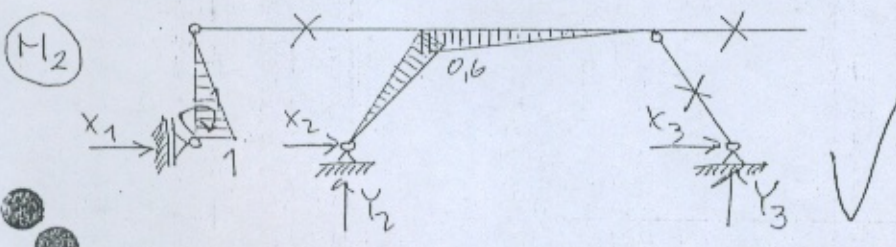
$$\boxed{X_1=0} \quad X_2+X_3=0 \Rightarrow \boxed{X_2=0.4}$$

$$Y_2+Y_3+1=0 \Rightarrow \boxed{Y_3=0.4}$$

$$1 \cdot 14 + Y_2 \cdot 10 = 0 \Rightarrow \boxed{Y_2=-1.4}$$

$$-Y_3 \cdot 2 - X_3 \cdot 3 = 0$$

$$-0.8 = 3X_3 \Rightarrow \boxed{X_3=-0.26}$$



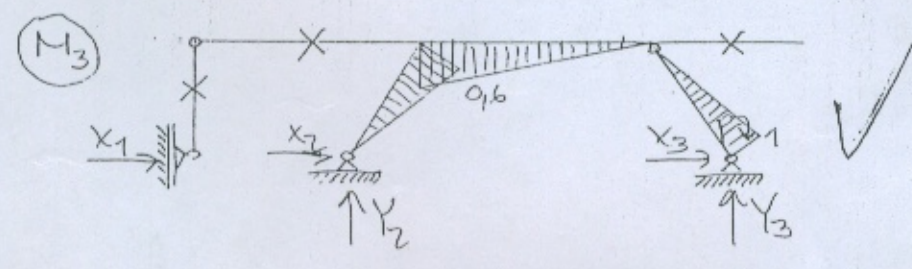
$$\boxed{X_1=\frac{1}{3}}$$

$$X_2+X_3+\frac{1}{3}=0 \Rightarrow \boxed{X_2=-\frac{4}{15}}$$

$$Y_2+Y_3=0 \Rightarrow \boxed{Y_3=\frac{1}{10}}$$

$$Y_2 \cdot 10 + \frac{1}{3} \cdot 3 = 0 \Rightarrow \boxed{Y_2=-\frac{1}{10}}$$

$$-Y_3 \cdot 2 - X_3 \cdot 3 = 0 \Rightarrow \boxed{X_3=-\frac{1}{15}}$$



$$\boxed{X_1=0}$$

$$X_2+X_3=0 \Rightarrow \boxed{X_2=-0.26}$$

$$Y_2+Y_3=0 \Rightarrow \boxed{Y_3=\frac{1}{10}}$$

$$Y_2 \cdot 10 + 1 = 0 \Rightarrow \boxed{Y_2=-\frac{1}{10}}$$

$$-Y_3 \cdot 2 - X_3 \cdot 3 + 1 = 0 \Rightarrow \boxed{X_3=0.26}$$

$$\delta_{ij} = \int \frac{M_i M_j}{EI} ds \Rightarrow \delta_{ij} EI_c = \frac{I_c}{I} \int M_i M_j ds \quad I_c = I$$

$$\delta_{11} EI_c = \frac{I_c}{I} \int M_1^2 ds = \frac{6}{3} 6^2 + \frac{\sqrt{13}}{3} 3.6^2 + \frac{1}{2} \frac{4}{3} 1.6^2 + \frac{1}{3} \frac{2}{3} (1.6^2 + 1.6 \cdot 2.4 + 2.4^2) \Rightarrow$$

$$\boxed{\delta_{11} EI_c = 91.98}$$

$$\delta_{22} EI_c = \frac{I_c}{I} \int M_2^2 ds = \frac{1}{15} \frac{3}{3} 1^2 + \frac{\sqrt{13}}{3} 0.6^2 + \frac{1}{2} \frac{4}{3} 0.4^2 + \frac{1}{3} \cdot \frac{2}{3} (0.4^2 + 0.4 \cdot 0.6 + 0.6^2) =$$

$$\boxed{\delta_{22} EI_c = 1.37}$$

$$\delta_{33} EI_c = \frac{I_c}{I} \int M_3^2 ds = \frac{1}{3} \frac{\sqrt{13}}{3} 1^2 + \frac{\sqrt{13}}{3} 0.6^2 + \frac{1}{2} \frac{4}{3} 0.4^2 + \frac{1}{3} \frac{2}{3} (0.4^2 + 0.4 \cdot 0.6 + 0.6^2) =$$

$$\boxed{\delta_{33} EI_c = 1.11}$$

$$\delta_{12} EI_c = \frac{I_c}{I} \int M_1 M_2 ds = -\frac{\sqrt{13}}{3} 3.6 \cdot 0.6 + \frac{1}{2} \frac{4}{3} 1.6 \cdot 0.4 + \frac{1}{3} \frac{2}{3} [1.6(2 \cdot 0.4 + 0.6) + 2.4(2 \cdot 0.6$$

$$\boxed{\delta_{12} EI_c = -1.50}$$

$$\delta_{13} EI_c = \frac{I_c}{I} \int M_1 M_3 ds = -\frac{\sqrt{13}}{3} 3.6 \cdot 0.6 + \frac{1}{2} \frac{4}{3} 1.6 \cdot 0.4 + \frac{1}{3} \frac{2}{3} (0.4^2 + 0.4 \cdot 0.6 + 0.6^2) \Rightarrow$$

$$\boxed{\delta_{13} EI_c = -2.32}$$



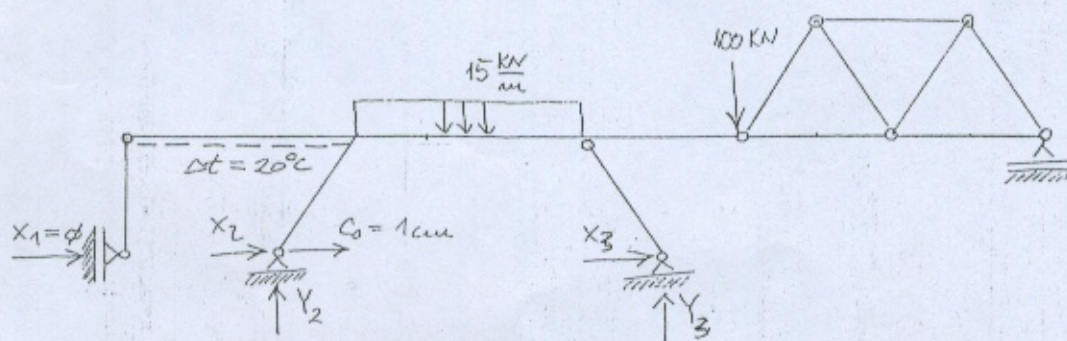
$$\boxed{\sum_{i=1}^3 EI_c = 0,71}$$

$$D = \begin{bmatrix} 91,98 & -1,50 & -2,32 \\ -1,50 & 1,37 & 0,71 \\ -2,32 & 0,71 & 1,11 \end{bmatrix} \frac{1}{EI}$$



(2.)

3

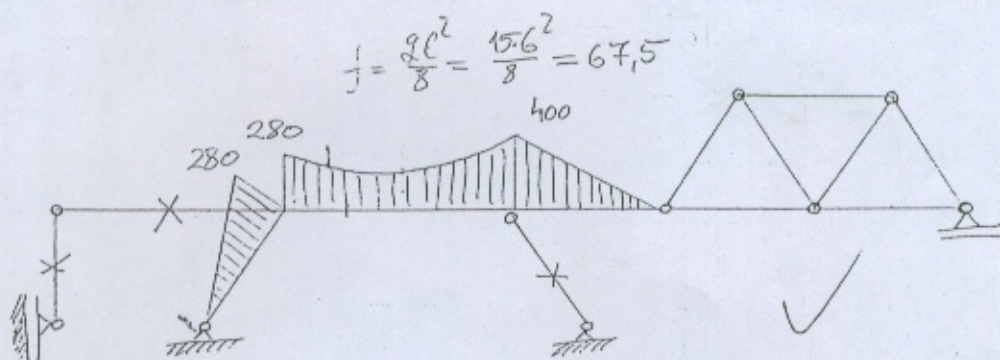


$$X_2 + X_3 = 0 \Rightarrow X_2 = 110$$

$$Y_2 + Y_3 - 100 - 90 = 0 \Rightarrow Y_2 = 25$$

$$-Y_3 \cdot 10 + 90 \cdot 5 + 100 \cdot 12 = 0 \Rightarrow Y_3 = 165$$

$$-X_3 \cdot 3 - Y_3 \cdot 2 = 0 \Rightarrow X_3 = -\frac{2}{3} Y_3 = -110$$



$M_0$

$$\delta_{i\phi} = \delta_{i0} + \delta_{it} + \delta_{ic}$$

$$EI_c \delta_{10} = \frac{I_c}{I} \int M_1 M_0 ds = \frac{\sqrt{13}}{3} 3,6 \cdot 280 - \frac{1}{2} \left( \frac{4}{3} 1,6 (230 + 400) - \frac{4}{3} 30 \cdot 1,6 \right) - \frac{1}{3} \left( \frac{2}{6} [2,4 (280 \cdot 2 + 230) + 1,6 (2 \cdot 230 + 280)] - \frac{2}{3} 7,5 \cdot (1,6 + 2,4) \right) = 209,21$$

$$EI_c \delta_{20} = \frac{I_c}{I} \int M_2 M_0 ds = -\frac{\sqrt{13}}{3} 9,6 \cdot 280 - \frac{1}{2} \left( \frac{4}{3} 9,4 (230 + 400) - \frac{4}{3} 30 \cdot 9,4 \right) - \frac{1}{3} \left( \frac{2}{6} [0,6 (2 \cdot 280 + 230) + 9,4 (2 \cdot 230 + 280)] - \frac{2}{3} 7,5 (0,6 + 9,4) \right) = -553,36$$

$$EI_c \delta_{30} = EI_c \delta_{10} = -553,36$$

24.02.2004.  $\checkmark$   
(40%)



$$EI_c \delta_{1c} = EI_c \cdot 1,2 \cdot \alpha + \frac{24}{L^3} d_1 = 10^5 \cdot 10^{-5} \frac{15}{1} \cdot 0 = 0$$

$$EI_c \delta_{3c} = EI_c \int M_3 \alpha + \frac{24}{L^3} d_3 = 10^5 \cdot 10^{-5} \frac{15}{1} \cdot 0 = 0$$

$$EI_c \bar{\delta}_{1c} = - \sum_j C_{j1} C_j = - 10^5 \cdot 0,01 \cdot 0,26 = -266,6$$

$$EI_c \bar{\delta}_{2c} = - \sum_j C_{j2} C_j = - 10^5 \cdot 0,01 \cdot 0,26 = -266,6$$

$$EI_c \bar{\delta}_{3c} = - \sum_j C_{j3} C_j = - 10^5 \cdot 0,01 \cdot (-0,26) = 266,6$$

$$\delta_{1\phi} = 209,24 + 270 - 266,6 = 212,58$$

$$\delta_{2\phi} = -553,36 + 0 - 266,6 = -820,02$$

$$\delta_{3\phi} = -553,36 + 0 + 266,6 = -286,7$$

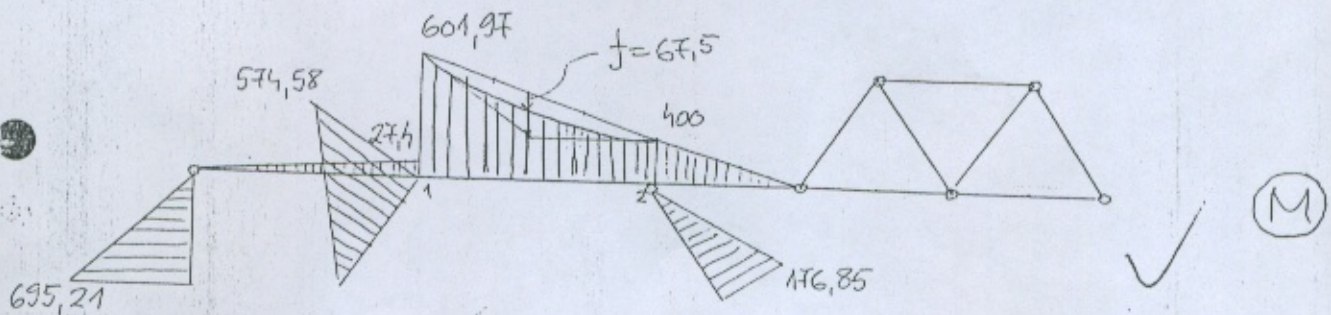
$$\begin{bmatrix} 91,98 & -1,50 & -2,32 \\ -1,50 & 1,37 & 0,71 \\ -2,32 & 0,71 & 1,11 \end{bmatrix} \cdot 10^{-5} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 212,58 \\ -820,02 \\ -286,7 \end{bmatrix} = 0$$

$$X_1 = -4,566$$

$$X_2 = -695,206$$

$$X_3 = 176,851$$

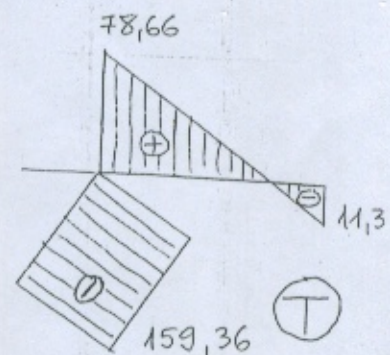
$$M = M_0 + M_1 X_1 + M_2 X_2 + M_3 X_3$$



$$T_{m1} = \frac{-400 + 601,97}{6} + \frac{15 \cdot 6}{2} = 78,66$$

$$T_{m2} = \frac{-400 + 601,97}{6} - \frac{15 \cdot 6}{2} = -11,34$$

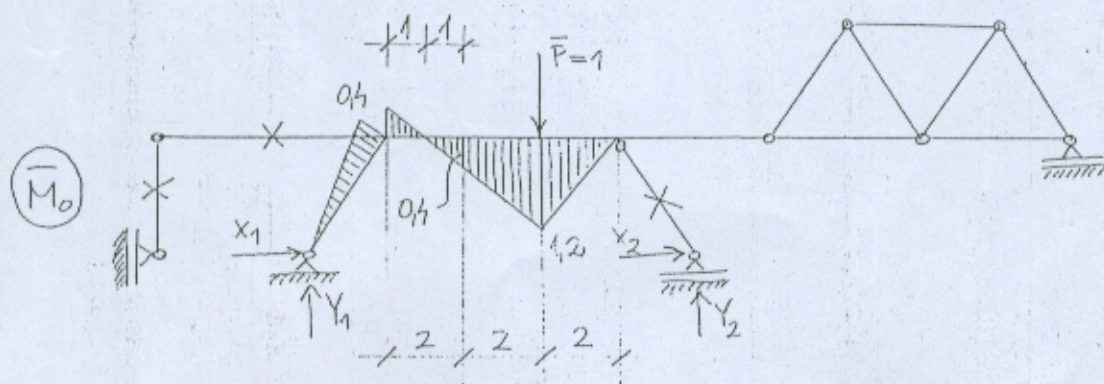
$$T_m = \frac{-574,58 + 0}{\sqrt{13}} = -159,36$$





(5.)

$$a) \theta_m = \int \frac{M \bar{M}_0}{EI} ds + \int \frac{N \bar{N}_0}{EF} ds + \int 1,6 \alpha t \frac{t}{2} ds + \int \bar{T}_0 \alpha t t^0 ds - \sum_j \bar{C}_j \xi_j$$



$$X_1 + X_2 = 0 \Rightarrow X_1 = 0,4$$

$$Y_1 + Y_2 - 1 = 0 \Rightarrow Y_2 = 0,6$$

$$Y_1 \cdot 10 - 1 \cdot 4 = 0 \Rightarrow Y_1 = 0,4$$

$$-Y_2 \cdot 2 - X_2 \cdot 3 = 0 \Rightarrow X_2 = -0,4$$

$$\begin{aligned} EI_c \theta_m &= \frac{I_c}{I} \int M \bar{M}_0 ds - \sum_j \bar{C}_j \xi_j EI_c = \frac{\sqrt{13}}{3} 574,58 \cdot 0,4 + \frac{1}{3} \left( \frac{2}{6} [0,4(2 \cdot 601,97 \right. \\ &\quad \left. + 474,64) - 0,4(2 \cdot 474,64 + 601,97)] - \frac{2}{3} 7,5(0,4 - 0,4) \right) + \\ &\quad + \frac{1}{2} \left( \frac{2}{6} [-0,4(2 \cdot 474,64 + 392,32) - 1,2(2 \cdot 392,32 + 474,64)] - \right. \\ &\quad \left. - \frac{2}{3} 7,5(-0,4 - 1,2) \right) + \frac{1}{2} \left( \frac{2}{6} [-1,2(2 \cdot 392,32 + 400) + 0] - \right. \\ &\quad \left. - \frac{2}{3} 7,5(-1,2) \right) - 10^5 \cdot 0,4 \cdot 0,01 = -778,72 \end{aligned}$$

b)

$$\begin{aligned} EI_c \delta_{10} &= \frac{\sqrt{13}}{3} 3,6 \cdot 0,4 + \frac{1}{3} \left( \frac{2}{6} (2,4(2 \cdot (-0,4) + 0,4) + 1,6 \cdot (2 \cdot 0,4 - 0,4)) \right) - \\ &\quad + \frac{1}{2} \frac{2}{6} (1,6(2 \cdot 0,4 + 1,2) + 0,8(2 \cdot 1,2 + 0,4)) + \frac{1}{2} \frac{2}{3} 0,8 \cdot 1,2 = \\ &= 2,92 \end{aligned}$$

$$\begin{aligned} EI_c \bar{\delta}_{20} &= EI_c \bar{\delta}_{30} = -\frac{\sqrt{13}}{3} 0,6 \cdot 0,4 + \frac{1}{3} \frac{2}{6} (0,6(2 \cdot (-0,4) + 0,4) + 0,4(2 \cdot 0,4 - 0,4)) - \\ &\quad + \frac{1}{2} \frac{2}{6} (0,4(2 \cdot 0,4 + 1,2) + 0,2(2 \cdot 1,2 + 0,4)) + \frac{1}{2} \frac{2}{3} 0,2 \cdot 1,2 = \\ &= 0,00934 \end{aligned}$$

$$EI_c \delta_{1\phi} = 2,92 - 266,6 + 270 = 6,32$$

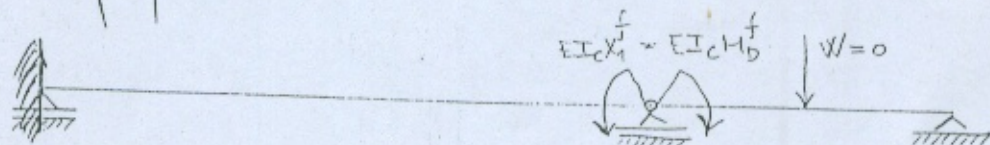
$$EI_c \bar{\delta}_{2\phi} = 0,00934 - 266,6 = -266,59$$

$$EI_c \bar{\delta}_{3\phi} = 0,00934 + 266,6 = 266,61$$

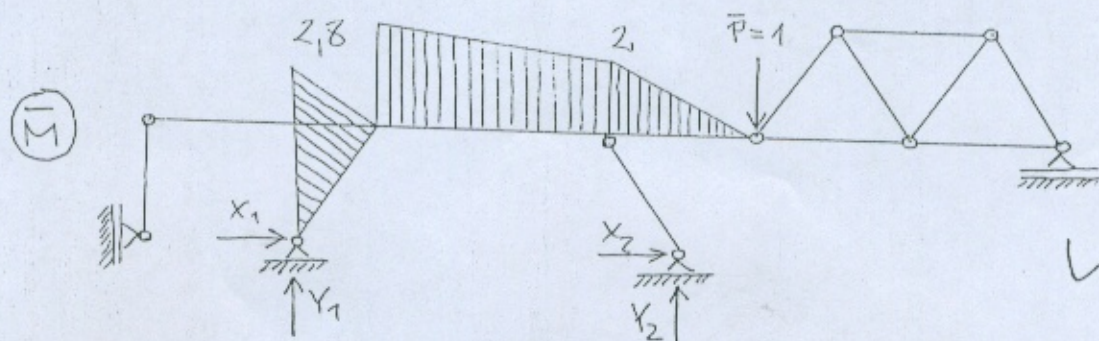


$$u = u_0 + u_1 x_1 + u_2 x_2 + u_3 x_3$$

поправ



ФУНКЦИОНАЛ  
НОСА



$$x_1 + x_2 = 0 \Rightarrow x_1 = 0.8$$

$$y_1 + y_2 - 1 = 0 \Rightarrow y_2 = 1.2$$

$$-2y_2 - x_2 \cdot 3 = 0 \Rightarrow x_2 = -0.8$$

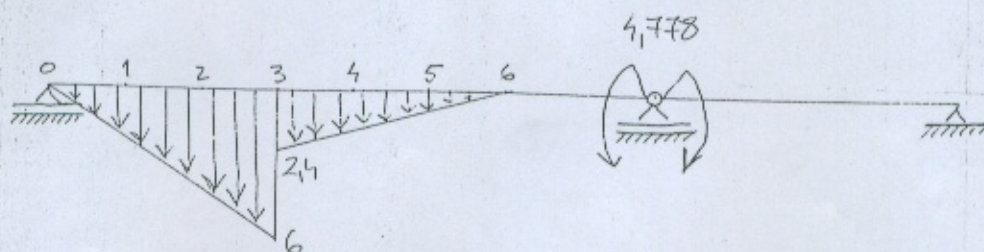
$$y_1 \cdot 10 + 1.2 = 0 \Rightarrow y_1 = -0.2$$

$$EI_c M_D^f = \frac{I_c}{I} \int M_1 \bar{M} ds = \frac{\sqrt{13}}{3} 2.8 \cdot 3.6 - \left( \frac{2}{6} [2.8(2 \cdot 2.4 + 1.6) + 2.53(2 \cdot 1.6 + 2.4)] \frac{1}{3} + \frac{4}{6} [1.6(2 \cdot 2.53 + 2) + 0] \frac{1}{2} \right) = 4.778 \quad (\text{за } x_1 = 1)$$

$$EI_c M_D^f = \frac{I_c}{I} \int M_2 \bar{M} ds = -\frac{\sqrt{13}}{3} 0.6 \cdot 2.8 - \left( \frac{2}{6} [2.8(2 \cdot 0.6 + 0.4) + 2.53(2 \cdot 0.4 + 0.6)] \frac{1}{3} + \frac{4}{6} [0.4(2 \cdot 2.53 + 2) + 0] \frac{1}{2} \right) = -3.853 \quad (\text{за } x_2 = 1)$$

$$EI_c M_D^f = \frac{I_c}{I} \int M_3 \bar{M} ds = -3.853 \quad (\text{за } x_3 = 1)$$

СТАЖЕ  $x_1 = 1$



$$P_0 = \frac{2}{6} (2 \cdot 0 + 2) = 0.6$$

$$P_1 = \frac{2}{6} (0 + 4 \cdot 2 + 4) = 4$$

$$P_2 = \frac{2}{6} (2 + 4 \cdot 4 + 6) = 8$$

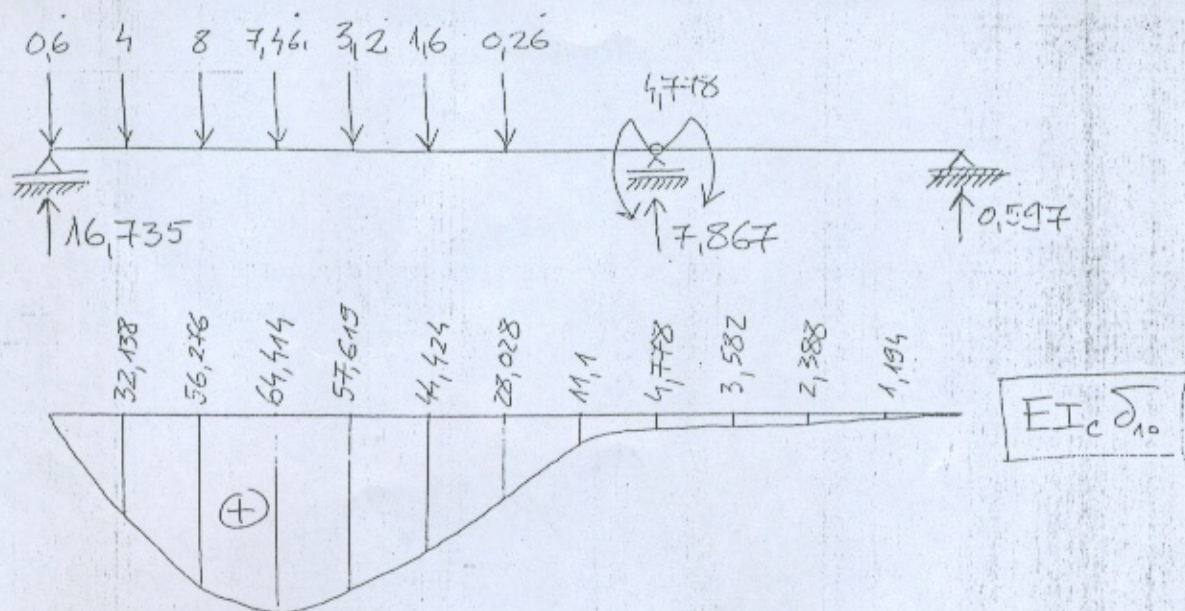
$$P_3 = \frac{2}{6} (4 + 2 \cdot 6) + \frac{2}{6} (2 \cdot 2.4 + 1.6) = 7.46$$

$$P_4 = \frac{2}{6} (2.4 + 4 \cdot 1.6 + 0.8) = 2.7$$

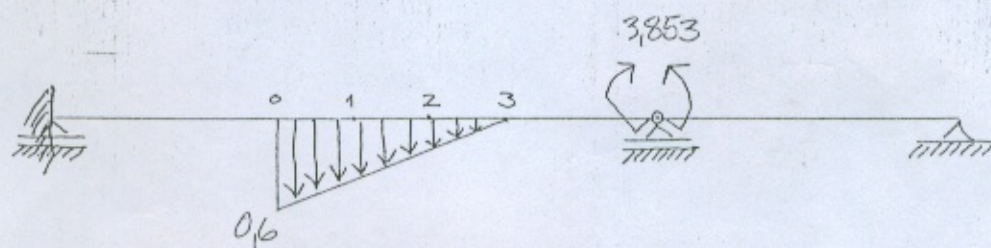
$$P_5 = \frac{2}{6} (1.6 + 4 \cdot 0.8 + 0) = 1.1$$

$$P_6 = \frac{2}{6} (0.8 + 0) = 0.26$$





СТАБЕ  $X_2 = 1$  и  $X_3 = 1$

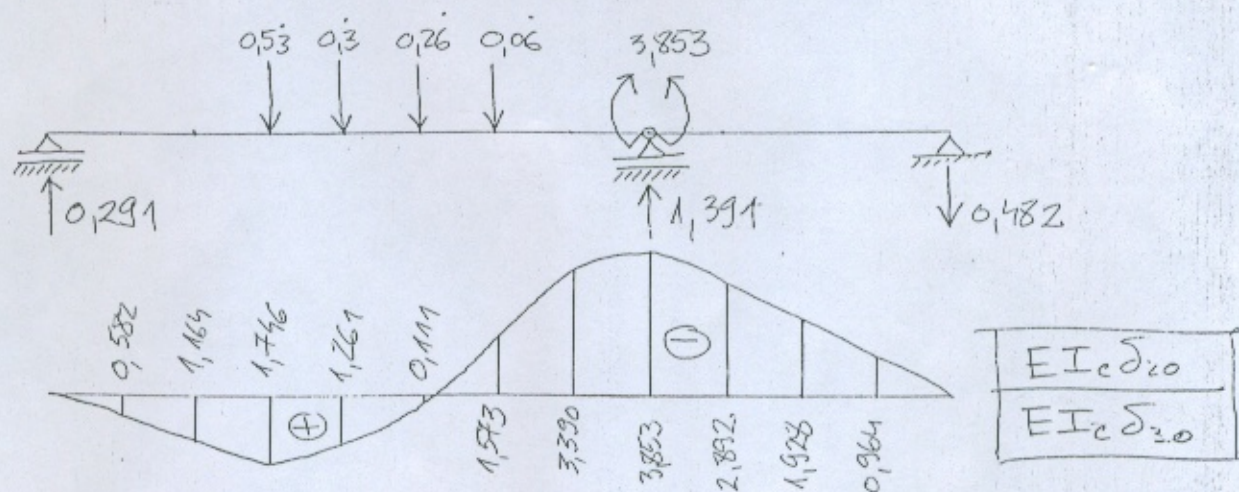


$$P_0 = \frac{2}{6}(2 \cdot 0.6 + 0.4) = 0.53$$

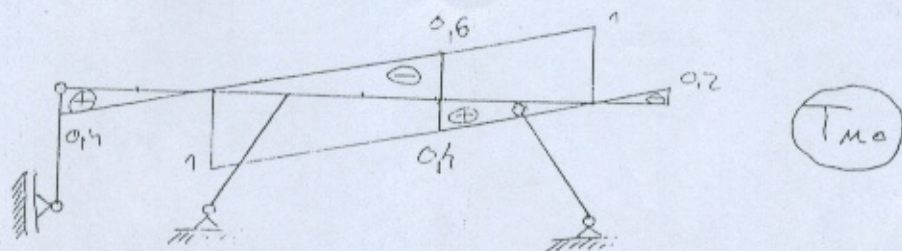
$$P_1 = \frac{2}{6}(0 + 2 \cdot 0.4 + 0.2) = 0.3$$

$$P_2 = \frac{2}{6}(0.4 + 2 \cdot 0.2 + 0) = 0.26$$

$$P_3 = \frac{2}{6}(0.2 + 0) = 0.06$$







$$T_{u1} = -0,4 \quad T_{u2} = -0,1 \quad T_{u3} = -0,1$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = - \begin{bmatrix} 0,0148 & 0,000295 & 0,0239 \\ 0,000295 & 1,692 & -0,698 \\ 0,0239 & -0,698 & 1,397 \end{bmatrix} \begin{bmatrix} EI \delta_{10}^+ \\ EI \delta_{20}^+ \\ EI \delta_{30}^+ \end{bmatrix}$$

$$T_u = T_{u0} - 0,4 \left( -0,0148 \delta_{10}^+ - 0,000295 \delta_{20}^+ - 0,0239 \delta_{30}^+ \right) -$$

$$- 0,1 \left( -0,000295 \delta_{10}^+ - 1,692 \delta_{20}^+ + 0,698 \delta_{30}^+ \right) -$$

$$- 0,1 \left( -0,0239 \delta_{10}^+ + 0,698 \delta_{20}^+ - 1,397 \delta_{30}^+ \right)$$

$$T_u = T_{u0} + 0,00834 \delta_{10}^+ + 0,040 \delta_{20}^+ + 0,795 \delta_{30}^+$$

i	0	1	2	3	4	5	6	7	8
$T_{u0}$	0,4	0,2	0	-0,2	-0,4	-0,6	0,2	0	-0,2
$\delta_{10}^+$	0	32,138	56,276	64,414	57,619	44,424	28,028	11,1	4,778
$\delta_{20}^+$	0	0,582	1,164	1,746	1,261	0,111	-1,573	-3,390	-3,853
$\delta_{30}^+$	0	0,582	1,164	1,746	1,261	0,111	-1,573	-3,390	-3,853
$T_u$	0,4	0,954	1,441	1,795	1,133	-0,137	-0,880	-2,738	-3,377



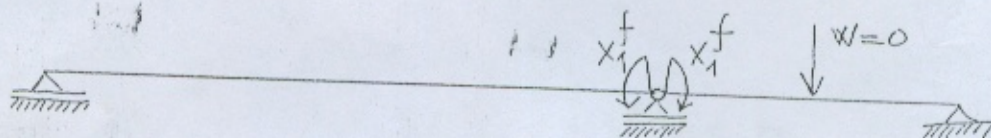
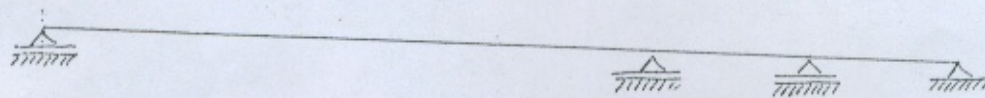
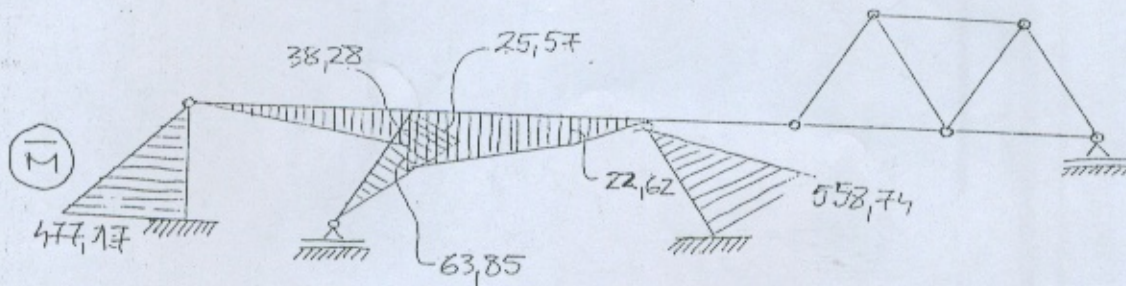
$$\begin{bmatrix} -1,5 & 1,37 & 0,71 \\ -2,32 & 0,71 & 1,11 \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \end{bmatrix} = \begin{bmatrix} -266,59 \\ 266,61 \end{bmatrix}$$

$$\bar{X}_1 = -6,38$$

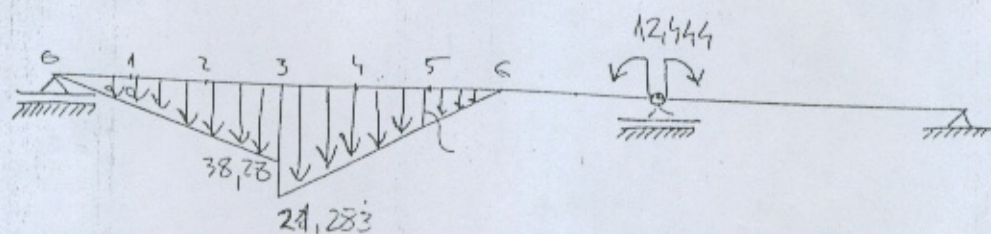
$$\bar{X}_2 = -477,17$$

$$\bar{X}_3 = 558,74$$

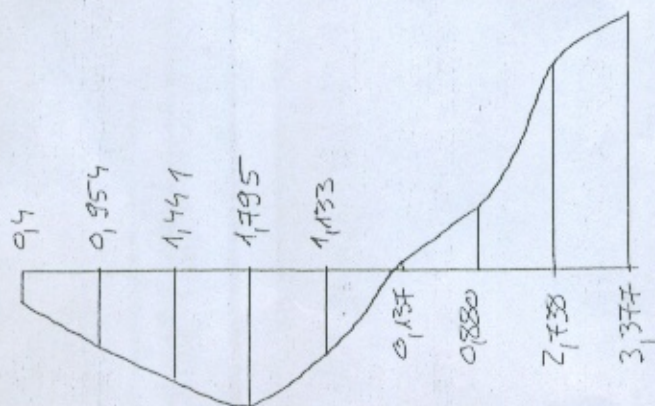
$$\bar{M} = \bar{M}_0 + M_1 \bar{X}_1 + M_2 \bar{X}_2 + M_3 \bar{X}_3$$



$$\begin{aligned} X_1^f &= \frac{I_c}{I} \int \bar{M}_0 M ds = -\frac{\sqrt{13}}{3} 25,57 \cdot 0,7 + \frac{1}{3} \frac{2}{6} [0,4 (2 \cdot (-63,85) - 43,235) \\ &+ (-0,4) (2 \cdot (-43,235) - 63,85)] + \frac{1}{3} \frac{2}{6} [-0,4 (2 \cdot (-43,235) - 22,62) \\ &+ (-1,2) (2 \cdot (-22,62) - 43,235)] + \frac{1}{2} \frac{2}{6} [-1,2 (2 \cdot (-22,62) + 0)] \\ &= 12,444 \end{aligned}$$







$T_m$

popravi!